Rigor, Relevance, & Best Practice Newsletter

Secondary MATH Edition

March 2015

Preparing Today's Students for Tomorrow's Challenges

Expressions & Equations Progression – A Look at Grades 6-8

The AZCCRS-Mathematics are composed of Domains that contain clusters of standards. All standards in the Expressions & Equations (EE) Domain have been designated as major Clusters in grades 6-8. Students in grades 6-8 will move through a progression of standards in expressions and equations that align with or closely resemble many of the standards taught in pre-Common Core Algebra I courses. *Traditionally, Algebra I started with the arithmetic and the simplifying of expressions, followed by solving of equations, then moving onto linear equations and systems by the end of first semester, with polynomials, quadratics and rational expressions rounding out second semester. AZCCRS-M spreads these concepts out over several years. Arithmetic, simplifying and basic solving is mastered in 6th grade. Solving multi-step equations and deeply understanding rates and ratios is the focus of 7th grade. The 8th grade standards focus on linear equations and systems. The key here is to see that the entire first semester of a traditional algebra course is covered by the end of 8th grade. This way, the students can be handed an exponential function when they walk in the door on the first day of their freshman Algebra class. Students are still learning Algebra in middle school; they are just not finishing it. The Common Core does not delay the Algebra course for students; it simply redefines Algebra.* (http://mathprojects.com/2013/06/06/common-core-pathways-redefining-algebra/)

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Expressions & Equations in Grade 6

Students understand the use of variables in mathematical expressions, write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. They understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. They construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

STANDARD	EXAMPLES
6.EE.A.1. Write and evaluate numerical expressions involving whole-number exponents.	***Write the following as a numerical expressions using exponential notation.*** *The area of a square with a side length of 8 m (Solution: $8^2 m^2$) *The volume of a cube with a side length of 5 ft.: (Solution: $5^3 ft^3$) *Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) ***Evaluate:*** * 4^3 (Solution: 64) * $5 + 2^4 \bullet 6$ (Solution: 101) * $7^2 - 24 \div 3 + 26$ (Solution: 67)

STANDARD	EXAMPLES
6.EE.A.2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*	Students read algebraic expressions in a manner that reinforces that the variable represents a number. • r + 21 as "some number plus 21 as well as "r plus 21" • n • 6 as "some number times 6 as well as "n times 6" • $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6" Students identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient).

Resources

The standards & examples in this publication were copied from the AZCCR-M documents at www.azed.gov. Here are a few links containing additional information, including details on what each child will be expected to know and do in each grade and tips for parents:

http://ccesa.az.gov/

www.azed.gov/AzMERIT

http://www.azed.gov/assessment/azsampleassessmentitems/

http://achievethecore.org/

https://www.engageny.org/

www.corestandards.org

www.pta.org/parentsguide

http://ww.azed.gov/standards-practices/files2012/05/rttt-implementation-plan-2-6-12.pdf

www.theteachingchannel.org/

STANDARD

6. EE.A.2. *continued*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

EXAMPLES

$x^2 + 5y + 3x + 6$

- The variables are x and y.
- There are 4 terms, x^2, 5y, 3x, and 6.
- There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively.
- The coefficient of x^2 is 1, since $x^2 = 1\ x^2$. The term 5y represent 5 y's or 5 * y.
- There is one constant term, 6.
- The expression shows a sum of all four terms.

Examples:

* 7 more than 3 times a number (Solution: $3x + 7$)
* 3 times the sum of a number and 5 (Solution: $3(x + 5)$
* 7 less than the product of 2 and a number (Solution: $2x - 7$)
* Twice the difference between a number and 5 (Solution: $2(z - 5)$
* Evaluate $5(n + 3) - 7n$, when $n = ½$.
* The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where *c* is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.

STANDARD

6. EE.A.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

EXAMPLES

* Interpret *3 (2 + x). For example, 3 groups of (2 + x).* Students use a model to represent x, and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x.*

An array with 3 columns and *x* + 2 in each column:

* Students interpret *y* as referring to one *y.* Thus, they can reason that one *y* plus one *y* plus one *y* **must be** *3y.* They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that

y + y + y = 3*y*: *y + y + y* = y x 1 + *y* x 1 + *y* x 1 = y x (1 + 1 + 1) = *y* x 3 = 3*y*

STANDARD

6. EE.A.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

EXAMPLES

Are the expressions equivalent? How do you know?

4m + 8 *4(m+2)* *3m + 8 + m* *2 + 2m + m + 6 + m*

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

STANDARD

6. EE.B.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

EXAMPLES

The equation 0.44*s* = 11, where *s* represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies you used to determine your answer. Show that your solution is correct using substitution.

Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?

STANDARD

6.EE.B.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

EXAMPLES

* An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: $28 + 0.35t$ where t represents the number of tickets purchased)
* Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned. (Solution: $15h + 20 = 85$ where h is the number of hours worked)
* Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item

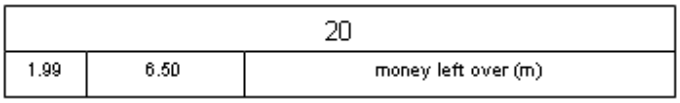
STANDARD

6.EE.B.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers

EXAMPLES

Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.

(Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$)

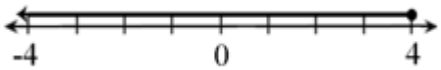


STANDARD

6.EE.B.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

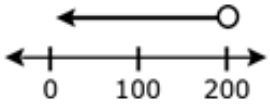
EXAMPLES

Graph $x \leq 4$.



Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.

Solution: $200 > x$



STANDARD

6.EE.C.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

EXAMPLES

Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table and graph to demonstrate the relationship between the numbers of weeks that pass and the amount in her savings account.

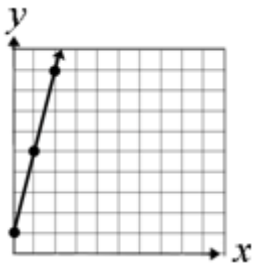
Language: Susan has $1 in her savings account. She is going to save $4 each week.

Equation: $y = 4x + 1$

Table:

x	y
0	1
1	5
2	9

Graph:



Expressions & Equations in Grade 7

In grade 7, students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts, students explain and interpret the rules operations with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

STANDARD

7.EE.A.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

EXAMPLE

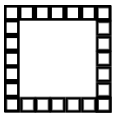
- Suzanne thinks the two expressions $2(3a - 2) + 4a$ and $10a - 2$ are equivalent? Is she correct? Explain.
- A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w + w + 2w + 2w$. Write the expression in two other ways. Solution: $6w$ OR $2(w) + 2(2w)$.
- An equilateral triangle has a perimeter of $6x + 15$. What is the length of each of the sides of the triangle? Solution: $3(2x + 5)$, therefore each side is $2x + 5$ units long.

STANDARD

7.EE.A.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

EXAMPLES

- Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way? Students may create several different expressions depending upon how they group the quantities in the problem.
- Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking.



STANDARD

7.EE.B.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

EXAMPLES

- The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

x	x	11
52		

$2x + 11 = 52$
$2x = 41$
$x = \$20.5$

STANDARD

7.EE.B.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

b. Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

EXAMPLES

- Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost?
- The sum of three consecutive even numbers is 48. What is the smallest of these numbers?
- Solve: $\frac{5}{4}n + 5 = 20$
- Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.
- Steven has $25 dollars. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy? Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution.
- Solve $\frac{1}{2}x + 3 > 2$ and graph your solution on a number line.

Expressions & Equations in Grade 8

In grade 8, students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). Additionally, students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

STANDARD

8.EE.A.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

EXAMPLES

- $\frac{4^3}{5^2} = \frac{64}{25}$
- $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$
- $\frac{4^{-3}}{5^2} = 4^{-3} \times \frac{1}{5^2} = \frac{1}{4^3} \times \frac{1}{5^2} = \frac{1}{64} \times \frac{1}{25} = \frac{1}{16,000}$

STANDARD

8.EE.A.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

EXAMPLES

- $3^2 = 9$ and $\sqrt{9} = \pm 3$
- $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$
- Solve $x^2 = 9$
  - $\sqrt{x^2} = \pm\sqrt{9}$
  - $x = \pm 3$
- Solve $x^3 = 8$
  - $\sqrt[3]{x^3} = \sqrt[3]{8}$
  - $x = 2$

STANDARD

8.EE.A.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

EXAMPLES

Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

STANDARD

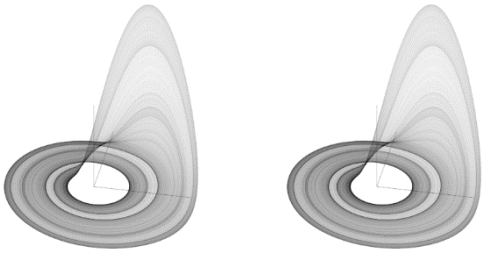
8.EE.A.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

EXAMPLES

Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of 2.45E+23 is 2.45 x 10^{23} and 3.5E-4 is 3.5 x 10^{-4}. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

STANDARD

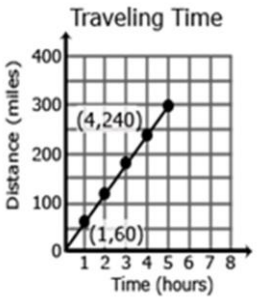
8.EE.B.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.



EXAMPLES

Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1:



Scenario 2:

$y = 50x$

x is time in hours

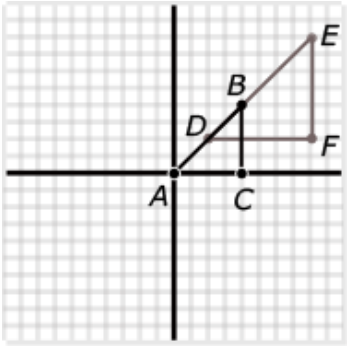
y is distance in miles

STANDARD

8.EE.B.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

EXAMPLES

Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation.



STANDARD

8.EE.C.7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

EXAMPLES

- Solve for x:

$$-3(x+7)=4$$

$$3x-8=4x-8$$

$$3(x+1)-5=3x-2$$

- Solve:

$$7(m-3)=7$$

$$\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$$

STANDARD

8.EE.C.8. Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

c. Solve real-world and mathematical problems leading to two linear equations in two variables.

EXAMPLES

- Find x and y using elimination and then using substitution.
  $3x + 4y = 7$
  $-2x + 8y = 10$
- Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

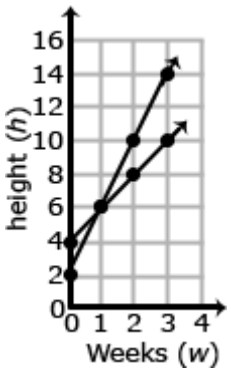
Let W = number of weeks
Let H = height of the plant after W weeks

Plant A		
W	H	
0	4	(0,4)
1	6	(1,6)
2	8	(2,8)
3	10	(3,10)

Plant B		
W	H	
0	2	(0,2)
1	6	(1,6)
2	10	(2,10)
3	14	(3,14)

- Given each set of coordinates, graph their corresponding lines.

Solution:



- Write an equation that represent the growth rate of Plant A and Plant B.

Solution:
Plant A $H = 2W + 4$
Plant B $H = 4W + 2$

- At which week will the plants have the same height?

Solution:
The plants have the same height after one week.

Plant A: $H = 2W + 4$	Plant B: $H = 4W + 2$
Plant A: $H = 2(1) + 4$	Plant B: $H = 4(1) + 2$
Plant A: $H = 6$	Plant B: $H = 6$

After one week, the height of Plant A and Plant B are both 6 inches.

Graphing Calculators in the Math Classroom

Within the Texas Instruments website you can download free activities that enhance your lesson plans and enable students to visualize mathematics. Visit the website at http://education.ti.com/en/84activitycentral/us/home. They have activities for Middle School Math, Algebra I, Geometry, Algebra II, Pre-calculus, Statistics and Calculus. All activities are linked to the current standards, have screen shots and tips for teachers.

Exponential Functions Activity – Guppies and Frogs

In this activity, students will create equations with two or more variables to represent relationships between quantities. They will also graph equations on coordinate axes with labels and scales.
As a result, students will:

- Write functions that describe a linear and exponential relationship between two quantities
- Interpret and compare the average rate of change of a function with the average percent change of a function
- Model real-world scenarios with equations and line graphs on a coordinate plane
- Analyze equations and graphs of linear and exponential models to determine properties of the models

Guppies and Frogs
Student Activity

Name _____
Class _____

A large pond contains 300 guppies and 160 frogs.

Suppose the guppy population increases by 25 guppies per year.

Suppose the frog population increases by 25% per year.

1. What does the future hold for this pond? Make a prediction.

2. What factors might impact the growth of the guppy and frog populations?

3. Use the words NOW and NEXT to write a rule showing how the guppy population changes each year. On the home screen of your calculator, simulate 6 years of guppy growth.

4. Let the function G represent the guppy population at year t. Complete the table.

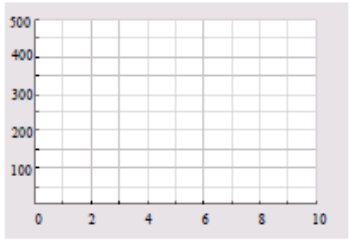
5. How does the guppy population change every 2 years?

Year, t	Guppies, G
0	
1	
2	
3	
4	
5	
6	

6. Graph the data (t, G) on the set of axes provided. Then, connect the data points.

7. Write a formula to model the population growth.

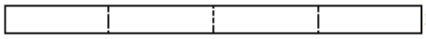
8. Use your formula to predict the population at year $t = 10$.



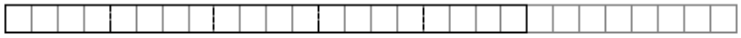
Guppies and Frogs
Student Activity

Name _____
Class _____

9. The initial population of 160 frogs grows by 25% each year. Suppose the rectangle below represents a population of 160 frogs.

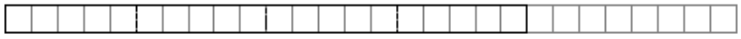


On the grid below, the total rectangle represents the population after the first year of 25% growth.



What is the population of frogs at year $t = 1$? _____

10. On the grid below, the model represents the population after 1 year. Use the grid to represent the population after 2 years.



What is the population of frogs at year $t = 2$? _____

Does the population increase by the same number of frogs each year? Explain your answer.

11. Use the words NOW and NEXT to write a rule showing how the frog population changes each year. On the home screen of your calculator, simulate 6 years of frog growth.

12. Interpret the meaning of the population at year $t = 3$ in terms of the context of the situation.

13. Let the function F represent the frog population at year t. Write a formula that models the population growth of frogs.

14. Use the table feature of your calculator to validate your formula. Draw the graph of the frog population on the set of axes on the previous page.

15. In what year does the population of frogs overtake the population of guppies?

16. List any limitations of these models for population growth.

Are you getting questions from parents about Arizona's College & Career Ready Standards? CCESA is offering FREE AZCCR-Math classes for parents on April 30 (K-12 Math Standards Overview), May 14 (K-5 Math Standards Focus), and May 18 (6-8 Math Standards Focus). All classes are FREE and will be held from 6-7pm at Ponderosa High School. Send parents to https://www.surveymonkey.com/s/ParentMathClass to register.